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IMPERIAL COLLEGE LONDON

Design Engineering MEng EXAMINATIONS 2023

For Internal Students of the Imperial College of Science, Technology and Medicine
This paper is also taken for the relevant examination for the Associateship or Diploma

DESE50002 – Electronics 2

Date: 3 May 2023 (one hour thirty minutes)

This paper contains 6 questions.
Attempt ALL questions.

SOLUTIONS

The numbers of marks shown by each question are for your guidance only; they indicate how the examiners intend to distribute the marks for this paper.

This is an CLOSED BOOK Examination.

1. a) For the signal $x(t)$ shown in *Figure Q1a*, and given that $u(t)$ is the unit step function, sketch each of the following signals in your answer book. The diagrams must show the values of $x(t)$ when $t = 0$.

   (i) $x(t)u(t)$

   (ii) $x(t)u(-t)$

   (iii) $x(t-2) - 1$

   (iv) $2x\left(\frac{t}{2} + 1\right)$

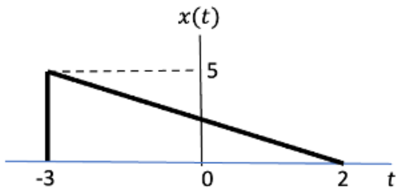
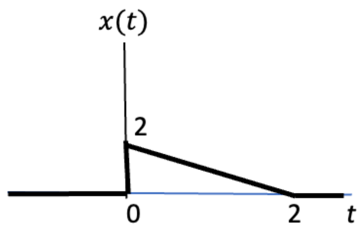


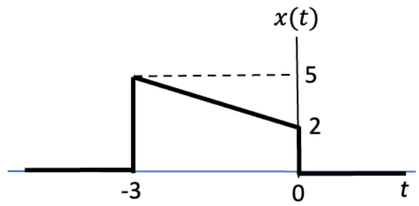
Figure Q1a

[8]

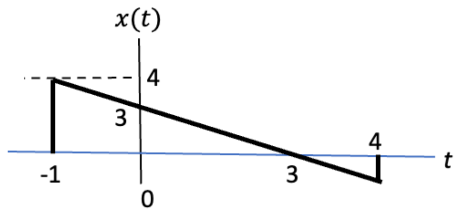
Q1 a) tests student's understanding on signal modelling and transform operation using shift, scale, stretch and compress.



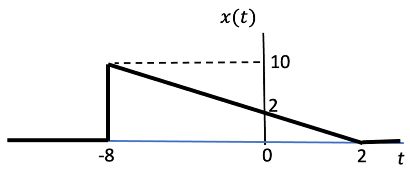
(i)



(ii)



(iii)



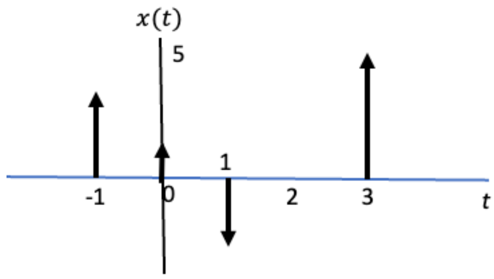
(iv)

b) A signal $x(t)$ is mathematically modelled by the following equation where $\delta(t)$ is the unit impulse function. Plot in your answer book the signal $x(t)$.

$$x(t) = 3\,\delta(t+1) + \delta(t) - 2\delta(t-1) + 5\delta(t-3)$$

[3]

Q1 b) tests student's understanding of the sampling property of the unity impulse function.



c) The power of a periodic signal $x(t)$ is given by the formula:

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)dt.$$

Derive from first principle the power of the signal if $x(t) = A\,cos(\omega_0 t + \theta)$.

[5]

What is the energy of the signal $x(t)$?

[2]

Q1 c) tests student's ability to derive stuff from first principle. This also tests students understanding the difference between power and energy of a signal.

(c)

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 \cos^2(\omega_0 t + \theta)dt$$

$$= \lim_{T\to\infty} \int_{-T/2}^{T/2} \frac{A^2}{2T} [1 + \cos\,(2\omega_0 t + 2\theta)]dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2T} \int_{-\frac{T}{2}}^{\frac{T}{2}} dt + \lim_{T\to\infty} \frac{A^2}{2T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos(2\omega_0 t + 2\theta)\,dt$$

The second term integrates to zero. Therefore:

$$P_x = \frac{A^2}{2}$$

The energy of the signal is infinite because it is an everlasting sinusoidal and not time limited.

2. a) A signal $y(t)$ is mathematically modelled by the following equation:

$$y(t) = 2.5\sin\left(200t + \frac{\pi}{6}\right) + 0.5$$

(i) What is the dc value, frequency in Hz, and phase in degrees, of $y(t)$?

[3]

(ii) Rewrite $y(t)$ in exponential form.

[5]

b) The Fourier series of a signal $f(t)$ is given by the equation:

$$f(t) = 16 + 12\cos\left(3t - \frac{\pi}{4}\right) + 8\cos\left(6t - \frac{\pi}{2}\right) + 4\cos\left(9t - \frac{\pi}{4}\right).$$

Sketch in the answer book the two-sided exponential Fourier spectrum $F(\omega)$ of the signal $f(t)$.

[9]

Q2 tests student's understanding of Euler's formula and Fourier spectrum derived from a time-domain mathematical representation.

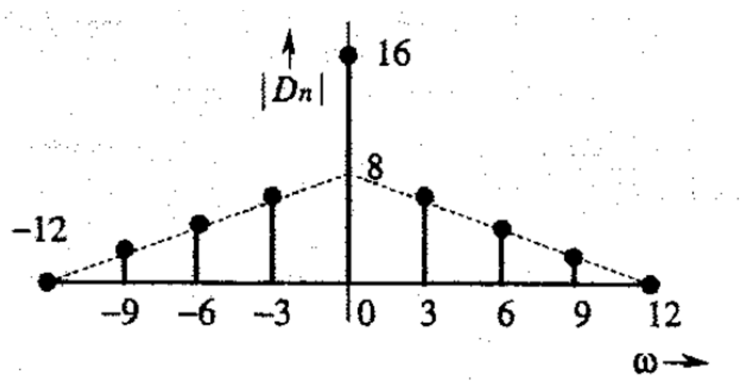
(a)

(i) DC value = 0.5V, Frequency = 31.83Hz, Phase = 30 degrees.

(ii)

$$y(t) = -1.25j\left(e^{j200t+\frac{\pi}{6}} - e^{-j200t-\frac{\pi}{6}}\right) + 0.5$$

(b)



3. Scientists discovered that the wing-flap frequency of mosquitoes for female and male are 400Hz and 600Hz respectively.

   a) A device is designed to detect mosquitoes by capturing in digital form the sound produced by mosquitoes' wing-flap with a sampling frequency of 2kHz is chosen. With justifications, comment on this design decision.

   [4]

   b) The device captures the wing-flap signal of a female and a male mosquito. Sketch the frequency spectrum of the captured signal between the frequency of 0Hz to 2kHz. You must show in your sketch the frequencies at which the signal has substantial energy, but with some arbitrary amplitude values.

   [6]

   c) Later, scientists discovered that when a male and a female mosquito conduct mating duet, their wing-flap frequency increases to around 1,200Hz.

   Sketch the frequency spectrum of the captured signal with two pairs of male-female mosquitoes when one pair is performing mating duet while the second pair is not. You should sketch the spectrum between the frequency of 0Hz to 2kHz.

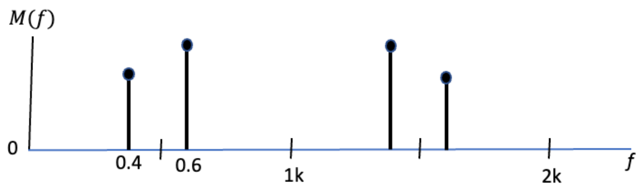
   [8]

Q3 tests student's understanding of sampling theorem, spectrum of a sampled signal, aliasing and frequency folding.

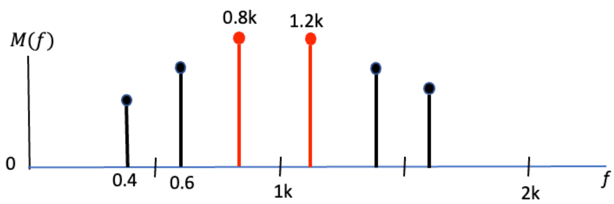
The reference for mosquitoes' wing-flap frequencies can be found here:
https://news.cornell.edu/stories/2009/01/study-mosquitoes-beat-out-love-song-mating

a) Sampling Theorem dictates that the minimum sampling frequency is 2 x 600Hz or 1,200Hz. 2,000 Hz is therefore a reasonable choice. It is sufficiently beyond the minimum sample frequency that no aliasing should occur. As can be seen later, this turns out not to be sufficient. Nevertheless, given the constraints provided in part a), the choice of sampling frequency is good.

b) Arbitrary amplitude values.



c)



4. The general transfer function of a simple second-order system is given by:

$$H(s) = \frac{{\omega_0}^2}{s^2+2\zeta\omega_0 s+{\omega_0}^2}.$$

The system in *Figure Q4* shows the block diagram of a second-order continuous-time system in the Laplace domain.

a) Derive the transfer function $H(s) = \frac{Y(s)}{X(s)}$ for the system.

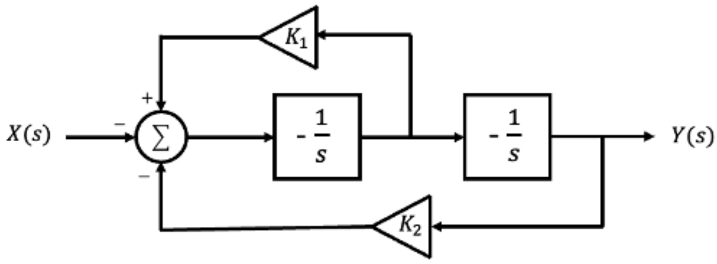
[5]

b) Using a), or otherwise, derive the relationship between input $x(t)$ and output $y(t)$ as a differential equation.

[5]

c) Given that $K_1 = 987$ and $K_2 = 44$, derive the DC gain, natural frequency in Hz and the damping factor of the system.

[5]



a)

$$H(s) = \frac{Y(s)}{X(s)} = -\frac{1}{s^2 + K_1 s + K_2}$$

b)

$$\frac{d^2y(t)}{dt^2} + K_1\frac{dy(t)}{dt} + K_2 y(t) = -x(t)$$

c) Since

$$H(s) = -\frac{1}{K_2}\left[\frac{K_2}{s^2+K_1 s+K_2}\right] = \frac{{\omega_0}^2}{s^2+2\zeta\omega_0 s+{\omega_0}^2}.$$

,

$K_2 = 44 = {\omega_0}^2 \;\; \omega_0 = 6.633\ \mathrm{rad/sec}$, hence resonant frequency is 1Hz.

$K_1 = 987 = 2\zeta\omega_0$, therefore, the damping ratio is 74.4.

The simplified equation given in the question as a dc gain of 1. However, this system has a dc gain of $-\frac{1}{K_2}$ = -0.227.

5. The z-domain transfer function H(z) of a first-order digital filter is given by:

$$H(z) = \frac{b_1 + b_2 z^{-1}}{1 + a_2 z^{-1}}$$

a) Sketch a function block diagram for this filter using constant multipliers, adder/subtractors and sample delay blocks.

b) Derive the difference equation for this digital filter.

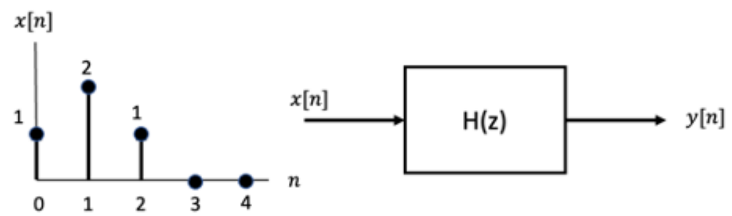
[6]

c) Given that $b_1 = 0.9, b_2 = 0$ and $a_2 = 0.1$, sketch the impulse response $h[n]$ of this filter for 0 ≤ n ≤ 3 .

[6]

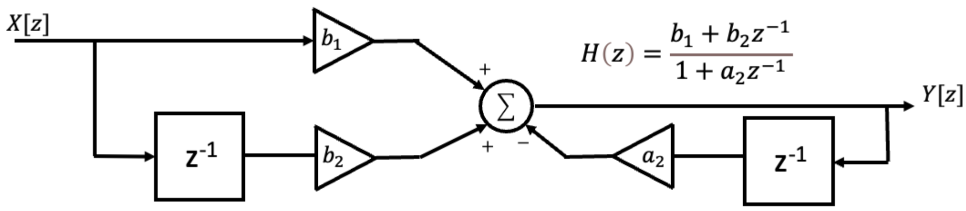
d) A causal signal $x[n]$ shown in Figure Q5 is applied to the input of the filter. By assuming that $h[n] = 0$ for $n \geq 4$, derive the output $y[n]$ for 0 ≤ n ≤ 5.

[8]



This question tests student's understanding of a discrete-time system and signal, z-transform representation of a digital IIR filter, and graphical method of convolution.

a)



[3]

b)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_1 + b_2 z^{-1}}{1 + a_2 z^{-1}}$$

$$Y(z)(1 + a_2 z^{-1}) = X(z)(b_1 + b_2 z^{-1})$$

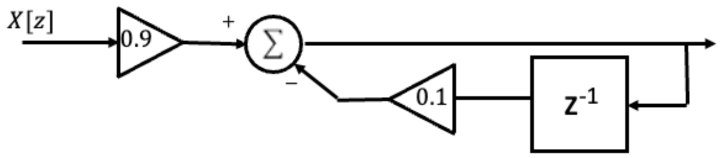
$$(Y(z) + a_2 Y(z) z^{-1}) = (b_1 X(z) + b_2 X(z) z^{-1})$$

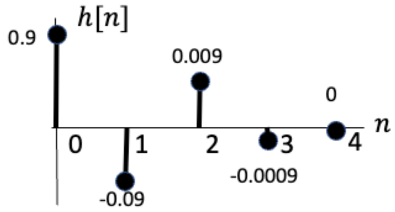
$$y[n] + a_2 y[n-1] = b_1 x[n] + b_2 x[n-1]$$

$$y[n] = b_1 x[n] + b_2 x[n-1] - a_2 y[n-1]$$

[3]

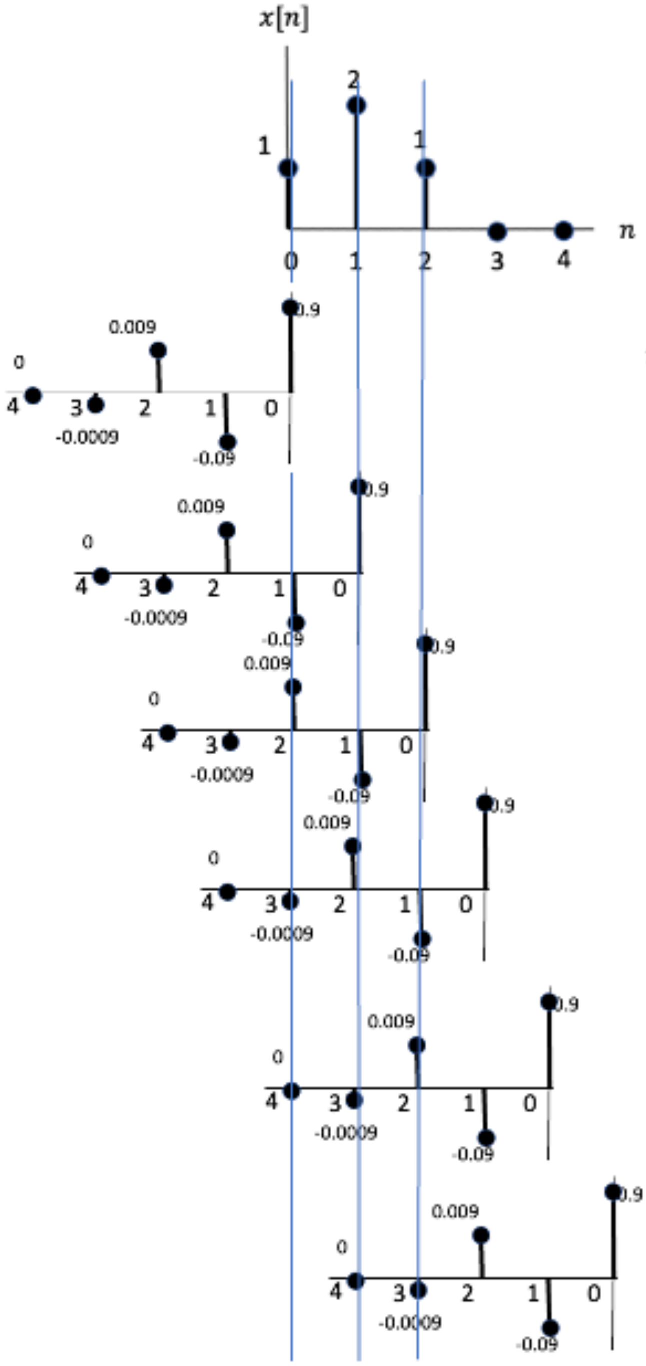
c)





[6]

d)



$n = 0 \qquad y[0] = 0.9$

$n = 1 \qquad y[0] = 1.8 - 0.09 = 1.71$

$n = 2 \qquad y[0] = 0.009 - 0.18 + 0.9 = 0.729$

$n = 3 \qquad y[0] = -0.0009 + 0.018 - 0.09 = -0.729$

$n = 4 \qquad y[0] = 0.0018 + 0.009 = 0.0072$

$n = 5 \qquad y[0] = -0.0009$

[8]

6. Figure Q6 show a simple proportional feedback system to control the motor speed $y(t)$ in response to the set-point $r(t)$ in the s-domain.

   a) Derive the closed loop transform function of the system $Y(s)/R(s)$. [4]

   b) Modify the system block diagram in Q6 to include a derivative term. [4]

   c) Explain the impact on the dynamic response of the system by adding a derivative term to the controller in the feedback system. [4]

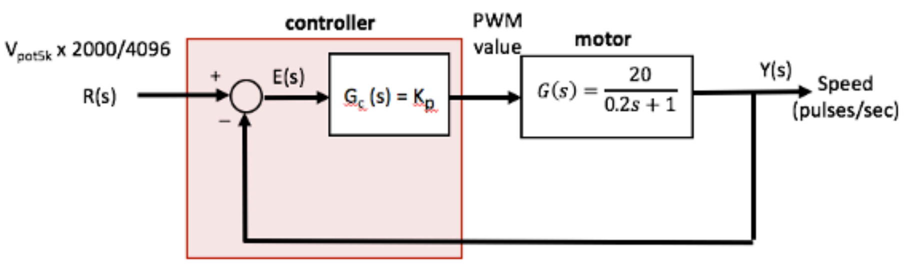
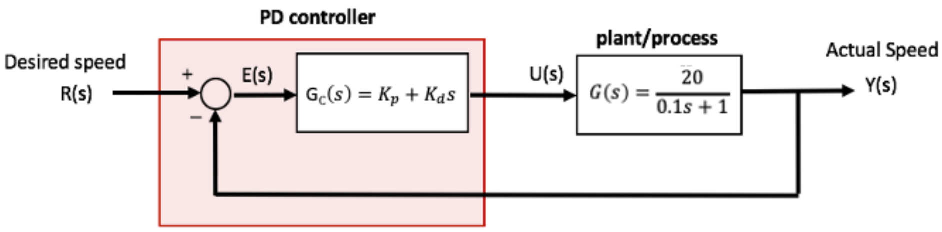


Figure Q6

Q6 tests student's understanding of basic feedback control and the PID controller.

(i) $H_{CL}(s) = \frac{20K_p}{1+20K_p+0.2s}$

(ii)



(iii) Bookwork. The derivative term acts as "brake" to the system. As the error E(s) is reduced, the derivative term is negative. This reduces the drive to the process (i.e. braking) and therefore prevents overshoot. The derivative term can also make the system more tolerant to external disturbances.